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THE MEASUREMENT OF LABOR PRODUCTIVITY

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Within the last five years a mass of writings has accumulated on the subject of labor turnover; more recently, absenteeism in labor has attracted attention and the two subjects have been recognized as closely connected. What has not yet been realized, however, is that parallel with these two "labor losses" goes a third, without which the usual statistical analysis and measurement of the efficiency of the personnel of an industrial plant is entirely lacking in final development. I refer to the possibility of an unduly low productivity on the part of the employee even when he is not "turning over" and not absent.

Probably this neglect of the final stage in ensuring personnel efficiency is due simply to the difficulty of measuring the degrees of loss involved. Labor turnover rates for the whole plant are now expressed in percentages of the average working force, and the hours lost by absenteeism are also quoted for the whole plant as a percentage of the total scheduled hours; but how can the actual degree of productivity in a multitude of different operations be a percentage of anything, or be quoted for a whole plant?

The question obviously is up to the statistician and until he solves the matter only the fringe of labor loss measurement is touched. For what is the use of a stable working force attending regularly if, possibly through very "staleness," the productivity of the individual man is low?

As in the case of labor turnover and absence, it is essential for the statistician to summarize the unit facts of any plant, or department, or operation, in such a way that the general showing of different plants, or different departments within a plant, or the different operations within a department can be compared fairly one with another. In fact, a coefficient is required of which the *numerator* gives the short-coming of the actual showing from the maximum showing considered possible and of which the *denominator* gives some definite measure of such possibilities. Each plant or department or single operation is thus judged by the deviation from the possibilities of its own situation.

In the case of labor turnover, the coefficient adopted has as its numerator the annual number of men leaving or the annual number of men replaced (there are two rival schools of thought) and as its denominator the average working force during the year. In the case of absence the numerator is the number of man-hours lost and the denominator the

total possible number of man-hours that could be worked according to schedule.

It is clear that the measure of absence is of the two the truer form of coefficient. The denominator gives the absolute possibilities which the numerator cannot exceed. In other words, the ratio can never have a greater value than 1, or 100 per cent. Moreover, the measure of *efficiency* (the attendance record) and the measure of *deficiency* (the absence record) are convertible, the one into the other. If Ab is the absence man-hours, and At the man-hours attended, and P the possible man-hours scheduled, $P = Ab$ plus At , and the absence coefficient $\frac{Ab}{P}$ can be derived from $\frac{P - At}{P}$.

The denominator of the turnover coefficient, on the other hand, gives only a rough indication of the absolute possibilities. We might assume that it was impossible for the whole working force of a plant to leave its employment more than once a day, supposing payment to be on a day-basis, not hourly. In that case the maximum possibility of turnover per year, assuming 300 working days per year, would be 300 times the working force, and the actual showing could be correctly expressed as a percentage of this. The average turnover has been found by the old method to have been in recent years around 200 per cent; stated as suggested, this would be equivalent to a coefficient of .66 per cent.

A LOW PRODUCTIVITY COEFFICIENT

Leaving aside the possible legitimization of the turnover coefficient on the lines of the absence coefficient, it is important that any *low productivity* coefficient that we may devise should start life in the right way. The possibilities of the situation in the matter of productivity (PP) must be known, and if the actual productivity (AP) is known, the coefficient of low productivity $\frac{LP}{PP}$ should be equivalent to

$$\frac{PP - AP}{PP} \text{ or } 1 - \frac{AP}{PP}.$$

How is the ratio of productivity $\frac{AP}{PP}$ to be found?

The objective unit in which productivity is measurable directly is the output; its quantity as so many identical pieces per hour, or if the work is not "repetitive" as a speed of so many minutes per job; and its quality as so much scrap per hundred pieces, or so many minutes wasted per hour.

Output is definitely recorded in industrial plants as part of the ordinary routine for the following purposes among others: to watch carefully the routing of the work, to ensure work up to specification, to determine the amount of the piece-wages to be paid, and also as a pure measure of the efficiency of the individual employees even when these are on a time wage.*

These records of productivity, whether quantity or quality of output is in question, may be placed as follows in ascending order of detail:

1. Records of the productivity per day of a gang† or squad of men on the same operation. We shall refer to these as group-day records.
2. Records of the productivity per day of individual men. We shall refer to these as man-day records.
3. Records of the productivity per hour of a squad or gang of men. We shall refer to these as group-hour records.
4. Records of the productivity per hour of individual men. We shall refer to these as man-hour records.

Records of types 1 and 2 are a usual part of plant bookkeeping, though the records are often destroyed after the lapse of three or four weeks. Records of type 3 are usually kept whenever scientific management has proceeded as far as the installing of a planning department, in order to schedule the routing and to follow the progress of the work. Records of type 4 are quite unusual as yet, and have to be specially demanded. Their collection, however, requires no disturbance of factory routine.

Certain indirect measures of productivity can also be used: for quantity of output the consumption of power in driving the machines, for quality of output the relative number of accidents. The consumption of power, be it noticed, only measures the output on machine work, while an accident, though undoubtedly the child of error in attention and faulty muscular coördination, just as is spoiled work, is by no means its twin brother.

The numerator of the ratio of productivity being manifest directly or indirectly in the actual showing of amount of output, speed of output, proportion of scrap, waste time and even consumption of power and accidents, how is the denominator indicating the degree of lowness of the productivity to be found? The word "low" gives us the clue. Lowness is a comparative word. The productivity must be low compared to productivity similarly manifested, but under other circum-

* H. L. Gantt writes: "If the conditions are such that we can plan out the work ahead of time, we will get a fair degree of efficiency by keeping individual records of the workmen and raising their day rate accordingly. As a matter of fact, a better efficiency can be obtained by this method than by the ordinary system of piece work, where the rates are set by past records or the estimates of the foreman."

† "Gang" is used to denote several men coöperating or combining on the same job; "squad" to denote several men working singly on similar jobs.

stances, and it is precisely some other "standard" production that must serve as our denominator.

How can a standard productivity be obtained?

Scientific management's time study will probably suggest itself immediately. Here is precisely a study whose main object is to find a humanly possible speed of operation that shall be a standard on which to calculate routing schedules, costs, and the piece rates premiums, or bonuses that are paid to the worker. The actual output is usually compared with this standard and a percentage efficiency worked out giving us, apparently, exactly the ratio of productivity we require.

But time-study is, so to speak, made *ad hoc*—specifically for one purpose, and it is often conducted only on one or two men during two or three short periods—probably too restricted a sample on which to judge the possible productivity of a whole group of men over their whole working time. The inadequacy of the time study in setting a standard productivity will be evident to practical statisticians when it is considered that those conducting these studies know nothing (or at any rate have published nothing) as to the relative productivity of the individual men and the particular time of day they select as samples, as compared with the possible productivity of the whole group of men and the whole working period. Time-study men do not seem even to be aware that there is such a thing as a normal curve of error in abilities. To them apparently all men are equal and all times are equal; there is no problem of sampling; the productivity of any man and any time will do to represent the standard productivity of the whole.

It must be conceded that "scientific managers" are vaguely conscious of shortcomings in their method and usually the standard speed set is some 20 per cent below the actual time study results in order to allow for human nature. This allowance saves the system from a *reductio ad absurdum*, but the whole procedure can hardly be called scientific.

It is the aim of the following study to find standards of productivity that are based on a sufficient number of samples and which, especially, are not based on an experiment introducing unusual elements, such as the stop-watch. The sample in fact must be taken from the ordinary plant records that are collected as part of the plant activities.

In fact we must use for our "standard" denominator precisely the same sources of information and the same plant records as we did for our numerator giving the actual showings. But these records—output records at any rate, as we have listed them—are of several different types: group-day records, man-day records, group-hour records, man-hour records. It would obviously be valueless to compare the

productivity of one man or group of men at one time with the productivity of another man or group of men at an entirely different time. The standard productivity must be of the same nature as the actual showing. If we wish to estimate the comparative productivity of different times, we must cite the productivity for different hours or days of the same man or group of men; if we wish to estimate the comparative productivity of different men, we must cite their productivity at one and the same periods of time. We thus reach the conclusion that, in the present state of our knowledge at any rate, the nature of the standard productivity must vary according to the problem we have in mind and the issues involved.

Here we shall confine ourselves to two specific comparisons that are both presenting important issues in industry today:

A. How does a man or a group or a squad of men compare in the various hours of the working day with its own standard for the most productive hour? What is the comparative *hour* productivity?

B. How does any given man's productivity in a squad compare with the productivity of all the men in the squad? What in fact is the *distribution of productivity* among the men in a squad? What is the comparative *man* productivity?

The importance to industry of an answer to question A lies in the problems of the daily cycle of loss of practice and fatigue. Do the earlier and later hours of the day show diminished productivity, and if so, what is the precise diminution say in the first hour, the eighth hour, the tenth hour? Does a ten-hour day pay? What length of working day is it that gives maximum productivity? And so on.

Question B has also certain very practical bearings in industry. Even supposing that one man's or a whole squad's productivity does not vary greatly from hour to hour, it does not follow that productivity is up to standard possibilities. While the course of output is smooth as between hours, the whole general level may be low.

In fact if comparative hour to hour productivity serves us as an index of daily fatigue, a comparison of individuals' general level of productivity probably may serve as an index of a chronic fatigue that is not removed by a night's rest. It may serve to point out easily fatigued individuals—neurasthenics—who would possibly form a class apart on a completely lower level of productivity.*

Further, the measure of comparative man-to-man productivity

*That industry contains enough subnormal individuals to constitute a severe loss is definitely maintained by psychologists and physiologists, as may be seen from Dr. Spaeth's articles in the *Journal of Industrial Hygiene*. Dr. Spaeth even goes so far as to assert that neurasthenics form such a large part in any group of workers that it is impossible to speak of any average typical productivity to include all the men in the group.

may throw light on a somewhat different kind of consideration—the voluntary restriction of output. For how else will this restriction manifest itself than by reducing the individuals most capable of high production to a level of man-productivity lower than their natural capacities would admit?

In the following paragraphs we shall illustrate the methods of computing coefficients of hourly productivity and man-productivity from figures supplied by a recently issued *Bulletin of the United States Public Health Service*.^{*} The main purpose of this *Bulletin* is the comparison of the working capacity of the personnel in two plants, one working a ten-hour day, the other an eight-hour day, but both engaged in metal working and both of equal technical efficiency.

HOURLY OPERATION OUTPUT

The standard performance for any one operation against which to measure the hourly productivity which was used in the *Public Health Bulletin* was the maximum output shown by any one of the eight or ten working hours of the day as found on a whole squad after at least a week's test.

The following table is drawn directly from the *Bulletin*,[†] and shows the result of such a test on four "squads," each engaged on a different

TABLE I: COMPARATIVE MAINTENANCE OF OUTPUT—

Operation	Number of observations	Starting time of shift	Period of study	(a) Number of units: (b) Index Numbers ¹	Average output observed each working hour	
					First	Second
Rivet press	64	7.30 a. m. . .	Oct. 13-30.	{ (a) (b) (a) (b) (a) (b) (a) (b)	630 91.2 246 96.1 435 92.4 763 85.2	663 95.9 253 98.8 471 100. 871 97.2
Roll coil	86	..do.do.			
Magneto taping	63 ⁴	..do.	July 23-Sept. 24. .			
Commutator loom	15 ⁴	..do.	Oct. 23-Nov. 15. .			
Average index numbers					91.2	98.0

¹ Index number=percentage variation from maximum; maximum=100.

⁴ Group days.

operation. Thus on the operation of rivet press, the third hour comes out as that of the maximum output and the actual average output per

^{*} Studies in Industrial Physiology: Fatigue in Relation to Working Capacity. Comparison of an Eight-Hour Plant and a Ten-Hour Plant. Report by Josephine Goldmark and Mary D. Hopkins on an Investigation by Philip Sargent Florence and Associates under the general direction of Frederic S. Lee. *Public Health Bulletin* No. 106.

[†] Table I, pp. 32 and 33.

man per day for this hour (691) is expressed as 100, the average output of all the other hours being then expressed as a percentage of this "standard" productivity.

This standard, as we learn from columns two and four, is based on 86 "observations," *i. e.*, 86 man-days, extending over a period of two weeks, and cannot, therefore, be considered a "snap" judgment. It is a result of conditions existing on twelve different workdays and affecting probably some seven or eight* different persons, and the chances are that everything has canceled out except the fact that the productivity was that of the third hour.

The last column but one, giving the average output for all the hours of the day, may be considered to sum up the comparative loss by low productivity for each operation. The roll coil operation, for instance, shows 96.5 per cent as the average of the hourly productivities over the whole day, which means that the low productivity coefficient $\frac{PP-AP}{PP}$ (possible "Standard" productivity minus actual productivity showing, divided by the possible standard productivity—see preceding page) is for this one operation $\frac{100-96.5}{100} = 3.5$ per cent, as measured on the hourly basis in the quantity of output.

EIGHT-HOUR PLANT, DEXTEROUS HANDWORK

Average output observed each working hour									Ratio of spells— second spell to first spell (in per cent)
Third	Fourth	Average for spell	Fifth	Sixth	Seventh	Eighth	Average for spell	Average for day	
691	² 654	660	653	642	651	³ 616	641	650	
100	94.6	95.5	94.5	92.9	94.2	89.1	92.8	94.1	97
256	² 252	252	236	252	253	³ 226	242	247	
100	98.4	98.4	92.2	98.4	98.8	88.3	94.5	96.5	96
451	² 448	451	456	455	452	³ 431	449	450	
95.7	95.1	95.7	96.8	96.6	96.0	91.5	95.3	95.5	99
871	² 896	850	856	845	835	³ 776	828	839	
97.2	100	94.9	95.5	94.3	93.2	86.6	92.4	93.6	97
98.2	97.0	96.1	94.8	95.6	95.6	88.9	93.8	94.9	97

² Period of $\frac{3}{4}$ of an hour; actual output multiplied by $\frac{4}{3}$.

³ Period of $1\frac{1}{4}$ hours; actual output divided by $\frac{5}{4}$.

HOURLY PLANT OUTPUT

We may give in this way the comparative hourly productivity of several squads on various individual operations, but how can we jump from this to a measurement of the hourly productivity of the entire

* The days are 12, the man-days 86. Men=86/12 on the average.

plant corresponding to the inclusive turnover and absence rates that are so freely quoted? The solution bears an obvious resemblance to the measurement of cost of living changes. There is the same difficulty of quoting one uniform figure for a multiplicity of different items—numbers of objects, weights, sizes, etc., and of so weighting the items that the composite whole will present a true picture of what is being measured.

The difficulty of unifying the multiform products is settled just as in the case of cost of living changes, by the index number; the base for this index number (to be represented by 100) lies ready to hand in the hour of maximum output, and the hourly variations in each of the operations studied are expressed as percentage variations from this hour. It will be noticed that this plan bears an analogy to the older method used by the United States Bureau of Labor Statistics in forming index numbers, namely, to express the changes in the price of each commodity by a separate series of index numbers of its own, before combining the commodity prices. The newer method of adding all prices together in their original form before indexing would in our case give undeserved weight to those operations which happen to produce the greatest number of units per hour. Thus in Table I on page 295, if the number of units were added in their original form, the commutator loom operation, where 839 units are produced per group per day on the average, would have over three times the weight of the productivity on the roll coil operation, where only 247 units per man happen to be produced on the average. Yet commutator loom is not necessarily the more important item in the production of the whole plant.

The use of the maximum hour as the basis for index numbers may meet criticism on the score that the maximum is by nature an extreme, and that the procedure is as inconvenient as basing *price* index numbers, say on the inflated prices of 1864, at the end of the Civil War. To this the authors of the *Bulletin* answer as follows:

“It is true that the use of the maximum is open to certain statistical objections. A [basis of] comparison must obviously not be subject to deviations of an extreme or violent character as a maximum may frequently be. It must offer some guaranty of a fair measure of stability. In the limitations of human working power such a guaranty is afforded in the present case. Spurts may for brief intervals occasion an extreme deviation, but since they do not in any operation last an entire hour, they do not raise any hour so greatly as to invalidate the use of the maximum as a standard.

“That our output figures are not subject to high extremes and are sufficiently massed toward the maximum to permit its use as the standard of comparison is proved by the following figures. The total number of average hourly outputs contained in the tables of both plants and represented in our text by the familiar term “index

numbers" is 344. Grouped according to their variation from the maximum 100, the index numbers are distributed as follows:*

	<i>Index Numbers</i>
From 100 to 96.	138
From 95 to 91.	112
From 90 to 86.	48
From 85 to 81.	18
From 80 to 76.	12
From 75 to 71.	6
From 70 and below.	9

"Thus a very high proportion—73 per cent—of these hourly output figures is massed within ten points of maximum; the remaining 27 per cent tapers down to the index number 48. The maximum then affords a satisfactory statistical basis of hourly comparison."

The difficulty now presents itself of so weighting the index numbers that, when added, their total (or average) will form a complete and accurately balanced picture of the whole plant productivity. In measuring cost of living changes, budgets are obtained of family consumption and commodity prices are weighted in accordance with the relative consumption of those commodities. In our case a "budget" must be obtained of the plant operations either through a census of occupations within the plant or, if the factory is small, by personal inspection. These various operations found must then be grouped according to what is considered the most important factor. In the case of a study of fatigue, the important question was held to be whether the operations involved muscle-work or dexterity, or whether it was a machine-operation. Accordingly, in the *Public Health Bulletin* which was mainly concerned with fatigue, the proportion of each type of operation in the whole plant is ascertained. The proportions of the eight-hour plant, which were made the basis for comparison, were as follows: one quarter of the operations, muscular; one quarter, dexterous; one half, machine operations, of which half again were operations on the lathe or "hand-screw" type of machine.

These proportions established, several operations of each of the four types were selected for study and an average derived from them for each type.† Table I, given on pp. 294–5, shows how an average is obtained from the four individual operations studied for the dexterous type at the eight-hour plant. Since each type forms exactly one quarter of all operations in the plant, the unweighted average of the four type-averages gives a representative average for the whole plant.

*For explanation of one additional index number not included here see p. 28 of the *Bulletin*.

† For the machine types an alternative plan is to use the power consumption records which would indicate variations in productivity in a uniform unit for all the different power-driven machinery.

The average "index number" for all the hours of the day on each type of operation in each factory is given below:

TABLE II: AVERAGE OF COMPARATIVE HOURLY PRODUCTIVITIES

Type of Work	Eight-Hour Plant	Ten-Hour Plant
Dexterous.....	94.9	91.6
Muscular.....	92.9	86.5
Lathe Machines.....	92.8	88.9
Miscellaneous Machines.....	95.3	94.0
Average for Whole Plant.....	94.0	90.3

As suggested above, these figures show the actual maintenance of productivity throughout the hours of the day and can form the numerator of the productivity ratio (AP). Averaging the four types of operation, the eight-hour plant is seen to have a low productivity coefficient $\left(100 - \frac{AP}{PP}\right)$ of $100 - \frac{94}{100} = 6$ per cent; the ten-hour a low productivity coefficient of $100 - \frac{90.3}{100} = 9.7$ per cent.

HOURLY PLANT ACCIDENTS

Besides the quantity of output, the *Public Health Bulletin* presents hourly comparisons in great detail for accidents. Just as a direct measure of quality of output would have to present the units spoiled or scrapped as a proportion of the total amount of units of output attempted, so here in the case of the indirect measure, the accidents have to be related to the total amount of production. The investigators were not satisfied with absolute accidents per hour, but give the comparative accidents per given output per hour. The ratio of accidents to output for each hour is arrived at by expressing each hour's accidents and total plant output as a percentage variation from the average hourly accidents and output respectively, and for each hour dividing the percentage accident variation by the percentage output variation.

The hourly total plant output used as a basis in calculating the hourly accident ratios lies, of course, ready to hand in the composite plant productivity described in the preceding section. Obtaining representative figures for the whole plant presents no difficulties either in the case of accidents. Where accidents are simply used as an indirect measure of the quality of output, the degree of seriousness of the accident is not significant. Few accidents are likely to have been purposely induced by the worker, and the severity of the injury thus controlled. In the majority of cases the seriousness of an accident is not correlated

definitely with the seriousness of the original error on the part of the human being (if any), and thus accidents are of uniform weight where human efficiency and low productivity are under study. The precise basis of the standard efficiency is decided by the purpose in hand. For the purpose of finding the effect of longer or shorter working hours, the basis for the standard minimum risk should be the lowest accident ratio shown on the average by any one particular working hour. This average in its turn should be based on a sufficient number of man-days.

What this standard is can obviously be found only from the factory records—motion or time-study on real accidents is decidedly out of the question.*

COMPARATIVE MAN-PRODUCTIVITIES

It has already been suggested that a second type of comparison, the comparison of man-productivities, would throw light on some further vital industrial issues—the possible existence of accumulated fatigue and nervous exhaustion of individuals, and the possible existence of restricted output.

It is evident that the basis for a solution of these issues is a knowledge of the distribution of working capacities among individuals. In the absence of positive evidence, we must assume that the working capacity of employees in any plant follows the normal curve of error which has been found to hold true of man's physical and mental traits. The distribution will be symmetrical about its mode, which is here equivalent to the arithmetic mean and median, and for any given coefficient of dispersion will assume a definite curvature.

Now the existence of the industrial phenomena we wish to study will be manifest in departures from this normal distribution of man-productivities. The presence of neurasthenics or the chronically fatigued will be ultimately shown in an unduly large frequency in the negative productivity values, possibly even in a second mode on the negative side of the curve, the neurasthenics definitely grouping themselves around a lower level of productivity. The existence of restrictions will be manifest by a departure downwards from the normal level of man-productivity among those of higher capacities, and there will be a sudden check in the progress of the curve at a certain level of productivity (the limit or "stint" generally observed). This non-correspondence with the normal curve of error will result (1) in the curve's being truncated at the higher values on the positive side, *i. e.*, negatively skewed, and (2) in the narrowing of the range of variation be-

* It is, of course, always possible to devise an apparatus which shall reproduce the supposed essentials in accident causation without reproducing its injurious effects. See, for instance, E. S. Bogardus, "The Relation of Fatigue to Industrial Accidents," in the *American Journal of Sociology*, 1911.

tween the more and the less productive, *i. e.*, a fall in the coefficient of dispersion. In the extreme form these results would amount to such a narrow distribution of values that one value would practically be stereotyped, and to avoid the unscientific emotions conjured up by the word "restriction," it is a wise plan to refer to this whole range of phenomena as "stereotyping."

Setting aside the effects of accumulated fatigue on comparative man-productivities for future consideration, we shall concentrate on devising a practical measure for the comparative loss involved by stereotyped production.

This measure to be practical must above all be intelligible and easily used by the ordinary employer of labor as also by the ordinary representative of labor.* We shall in consequence use the system of statistical measurements most easily grasped and most easily calculated, namely the Median-Quartile-Interquartile Range Group. This system of measurement has an additional advantage in that it discounts the extreme values which in the complex situation presented by industry occur so frequently for extraneous reasons.

For instance, in the following table, the lowest average output is some 124 pieces below the next lowest, whereas the interval between all other man-productivities is nowhere greater than 6.3 pieces. It is probable that some special condition is present, personal to this one low-producer without reference to general plant or operation conditions. An average of standard for this whole operation should not be unduly affected by it.

The theoretical disadvantage of the median as an average,† that a correct total cannot be obtained by multiplying the median by the number of items, is of minor importance if we are willing to waive absolute correctness in favor of general tendencies more likely to be correct in the long run. And absolute correctness is not so important where our main object is to measure the comparative efficiency or rather deficiency of different operations, departments or plants, as long as the same method of measuring is adopted in all cases compared. Further, in calculating his labor turnover, it does not matter much to an employer whether his rate is 120 per cent or 150 per cent per year, if his rivals have rates varying from 40 per cent to 80 per cent. In the present state of the science of personnel, we must be content if we can get but one step further in our comparisons than just "bad," "moderate," "fair," "good," "very good."

It was suggested that restriction of productivity will manifest itself

* One of the hoped for functions of a system of labor statistics in the plant is to form a basis for negotiation which both sides will be ready to accept as fact.

† The average is here used as the generic term to cover arithmetic mean, mode, median, etc.

in reducing the more capable individuals to a level of output lower than would "naturally be," *i. e.*, lower than a normal curve of error would admit. Let us make the conservative assumption that the productivity of the least productive half of the men is not affected by restriction. In other words, the average productivity of the lower half, *i. e.*, the lower quartile, remains the same.

We then have to find the average productivity of the most productive half of the men compared with that of the least productive half, *i. e.*, the percentage the upper quartile bears to the lower quartile, (A) under conditions of restriction, (B) under conditions where normal abilities have free play, the difference in the A and B percentages marking the loss in productivity due to restriction.

We can illustrate this method from tables published in the *Public Health Bulletin* (Chapter 4 "Stereotyped or Restricted Output") in which the average output of each man of a large squad is given for two similar operations, one at the ten-hour plant, the other at the eight-hour plant. Unless the operations are really of similar type, it is difficult to eliminate the tendency of the mechanical equipment to restrict output automatically. Thus if in one operation half the time is necessarily consumed by the actual contact of tools and material and this time cannot in any way be reduced by the worker, it follows that the possible increase in speed for the whole operation is physically restricted. Physical restriction will produce exactly the same divergence from the normal expected distribution of man-productivity as would voluntary restriction. Comparison of such a type of operation with, say, a hand operation, would fail to isolate the voluntary restriction.

With this caution in mind let us take the average outputs for the period December 31, 1917-January 17, 1918, of the twenty-one men "forming small end of base on lathe." These figures are given in Table 15 of the *Bulletin*. Arranged in order of magnitude the figures are as follows:

TABLE III: TEN-HOUR PLANT, OUTPUT OF STEREOTYPED OPERATION

FORM SMALL END OF FUSE ON LATHE

Order of Low Productivity	Check Number of Man	Average Output per Day Dec. 31, 1917-Jan. 17, 1918	Order of Low Productivity	Check Number of Man	Average Output per Day Dec. 31, 1917-Jan. 17, 1918
1st.....	10484.....	700.4	12th.....	10366.....	836.2
2nd.....	10420.....	824.1	13th.....	10361.....	836.9
3rd.....	10493.....	830.2	14th.....	10367.....	836.9
4th.....	10358.....	831.0	15th.....	10560.....	837.0
5th.....	10468.....	832.5	16th.....	10547.....	837.3
6th.....	10353.....	833.1	17th.....	10496.....	837.9
7th.....	10551.....	833.4	18th.....	10440.....	839.1
8th.....	10365.....	833.4	19th.....	10384.....	840.4
9th.....	10355.....	835.0	20th.....	10510.....	846.7
10th.....	10489.....	835.3	21st.....	10508.....	851.1
11th.....	10505.....	835.5			

The value of the lower quartile is the output of a worker coming, to be exact, fifth and three quarters in order.* If we wish to simplify matters as far as possible—an important consideration when appealing to business men—we may take the nearest *actual* worker, namely, the sixth in order. This is what was done in the *Bulletin* we are quoting from. The sixth worker has an average output of 833.1 fuses per day. By similar methods it was found that the value of the upper quartile (the output of the sixteenth worker on our list) was 837.3.

Now the question arises, what would have been the upper quartiles had the output not been stereotyped by any individual worker or, at any rate, not by one half of the individuals. Let us turn to the fullest example given in the *Public Health Bulletin* of a non-stereotyped operation of a similar nature, namely, "axle grinding on lathe," at the eight-hour plant (Table 17). Arranged in order of magnitude, the average daily outputs of the twenty-nine individual workers over the period studied were as shown in Table IV.

The quartile values quoted here are the output of the workers coming nearest to seven and three-quarters and twenty-two and a quarter, *i. e.*, the eighth and the twenty-second. These values are 382 and 408 respectively, the upper quartile being thus 106.806 per cent of the lower quartile.

In a non-stereotyped distribution of man-productivities on a lathe type of operation, then, the upper quartile may be expected to be 6.806 per cent above the lower quartile. Applying this expectation to the forming small end of fuze of lathe, the expected upper quartile

* In calculating the position of the median and quartiles in a series of n items, the author has not used the orthodox formulae (as given for instance by King in his "Elements of Statistical Method," Edition of 1916) to wit: $\frac{n \text{ plus } 1}{2}$, $\frac{n \text{ plus } 1}{4}$ and $\frac{3(n \text{ plus } 1)}{4}$ When worked out in spaces along a line, it will be found that these formulae make the interquartile range enclose slightly more than half the total spaces. Thus for 21 item-spaces Q_1 falls at $5\frac{1}{2}$, *i. e.*, exactly between the fifth and sixth spaces. Q_3 falls at $16\frac{1}{2}$, *i. e.*, exactly between the sixteenth and seventeenth spaces, and the interquartile range encloses 11 complete spaces out of the 21 instead of $10\frac{1}{2}$ as it should.

The formulae used by the author, which appear to be spatially correct, are as follows:

$$\text{Median: } \frac{n}{2} \text{ plus } \frac{1}{2}$$

$$\text{Lower Quartile: } \frac{n}{4} \text{ plus } \frac{1}{2}$$

$$\text{Upper Quartile: } \frac{3n}{4} \text{ plus } \frac{1}{2}$$

In the case of the median, the author's formula gives the same result as the orthodox formula, but the position or rank of the lower quartile is raised by $\frac{1}{2}$ and the position or rank of the upper quartile reduced by $\frac{1}{2}$.

The difficulty has arisen owing to our practice of indicating any space along a line in language by the position of its further boundary point, which, actually, is half the space beyond the center of the space. Thus we speak of a child's being in his fourth year when actually his age is anywhere between three and four and averages three and a half. Hence, to indicate in language any space in an array we must always add half a space to the actual middle of that space.

TABLE IV: EIGHT-HOUR PLANT, OUTPUT OF NON-STEREOTYPED OPERATION

AXLE GRINDING ON LATHE					
		Average Output per Day (Period Studied was Sept. 19-Oct. 13 and Oct. 29-Nov. 10, 1917)			Average Output per Day (Period Studied was Sept. 19-Oct. 13 and Oct. 29-Nov. 10, 1917)
Order of Low Productivity	Check Number of Man		Order of Low Productivity	Check Number of Man	
1st.	4238.	307	16th.	4452.	399
2nd.	4287.	347	17th.	4220.	400
3rd.	4206.	370	18th.	4465.	401
4th.	4282.	374	19th.	4361.	402
5th.	4342.	377	20th.	4289.	406
6th.	4182.	380	21st.	4419.	407
7th.	4003.	381	22d.	4395.	408
8th.	4373.	382	23d.	4318.	410
9th.	4370.	387	24th.	4392.	411
10th.	4354.	390	25th.	4200.	413
11th.	4402.	391	26th.	4221.	414
12th.	4215.	392	27th.	4232.	416
13th.	4276.	392	28th.	4426.	425
14th.	4223.	394	29th.	4350.	428
15th.	4356.	398			

should on that operation be $833.1 \times \frac{106.806}{100.00} = 889.801$. Actually, how-

ever we found the upper quartile to be 837.3.

The most productive half of the squad on this "form small end of fuze" operation, therefore, lost. $889.8 - 837.3$ fuzes per day by restriction, *i. e.*, 52.5. Since, however, we assumed the least productive half to lose nothing, the average loss over the whole group is $\frac{52.5}{2} = 26.25$ fuzes per man for the whole squad.

The net average loss in productivity for the whole squad, calculated (according to the formula suggested above) in percentages of the expected upper quartile (the standard probability) is

$$\frac{1}{2} \left\{ \frac{\text{Expected upper quartile} - \text{actual upper quartile}}{\text{Expected upper quartile}} \right\} = 2.95 \text{ per cent.}^*$$

ENVOI

In conclusion a word must be said as to the value to sociology of industrial statistics of the type we have outlined. The term sociology is used not as a vague reference to general social righteousness, but in its etymologically correct sense of the scientific *study* of society. It is a common observation that even the calmest students of mankind are likely, when they reach our modern industrial organization, to go wild and to forsake the path of ordinary scientific process for a jungle of vast overshadowing "problems"—the "labor problem," the "trust problem," the "railroad problem." The setting of problems to one's pupils or to oneself is a sure method of progression, provided the problem can be definitely formulated behind a question mark;

* *I. e.*, $\frac{26.25}{889.80}$.

but what exactly is THE question about labor, about trusts, about railroads? A treatise on modern industrial society seems either to confess a wholly perplexed state of mind or else to indicate a reversion to Euclidian geometrics. Probably both elements are present. The sociologist who had been content to describe the manorial economy, the organization of the town gilds, or the arts and crafts of the Senegalese, by piecing together his facts item by item, when he comes to existing conditions abandons the item as a basis and reverses the procedure. He starts from certain general ideas of what is, and when he strikes an item-fact that deviates therefrom, it is a "problem." And general ideas of what *is* are easily polluted by general ideas of what *should be*; hence any and every item that is simply wrong is a "problem," and the erstwhile sociologist is by this time a sort of Big Brother to us all as well as a deductive geometrist.

The only escape from the problem jungle is to go back to the item facts. It is true that modern society presents a plethora of items. Indeed we suspect that some brand of agarophobia was partly responsible for the resort to the "Problem" But it is just against this overwhelming multitude of item-facts that statistics comes to the sociologist's aid. Whenever the items have values that can be measured, the possibility of exact yet inclusive generalization is admitted.

Probably of all forms of modern society the industrial establishment or "plant" presents the greatest chance of collecting measurable items. What is it but its rigid standardization that is most complained of in modern industry? Yet is it not precisely standardization of conditions that social statistics sighs for? Once let the personnel of the industrial plant be regarded not as a tediously refractory part of an automatic mechanism, but as a social organism just as worthy of scientific treatment as the State or the Trade Union; once let the possibilities of a statistical study of its behavior be thoroughly explored, and more light will be shed on the reactions of mankind to its present admittedly difficult predicament than by an infinite solution of problems to their last Q. E. F.